Propagating Spacecraft Trajectory by Δt

Aubrey Lane Baker

Methods

➢ **Analytical**

➢ **Numerical Integration**

Analytical Method

Energy Equation (Vis-Viva eq.)

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

Energy [LU^2/TU^2], Initial Velocity [LU/TU], Escape Velocity [LU/TU]

```
energy_elps =
    -0.101950127000191

energy_parab =
    0

energy_hyp =
    0.158750158750238
```

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

```
V0Mag_elps =
    0.223830292855994

V0Mag_parab =
    0.503984378726544

V0Mag_hyp =

    0.755976568089817
```

```
Vesc_elps =
    0.503984378726544

Vesc_parab =
    0.503984378726544

Vesc_hyp =
    0.503984378726544
```

Eccentricity and Angular Momentum [LU^2 * MU / TU]

eMag_elps =
    0.6809713224416442

eMag_parab =
    1

eMag_hyp =
    1.380510539273007

hMag_elps =
    1.621758305050417

hMag_parab =
    1.126043215805762

hMag_hyp =
    1.689064823708642

TYPE ORBIT	Circular[1]	Elliptical	Parabolic[2]	Hyperbolic
Eccentricity, e	e = 0	0 < e < 1	e = 1	e > 1

Initial True Anomaly [rad], Initial time past periapse [TU], Final True Anomaly [rad], and Delta True Anomaly [rad]

```
nu0_elps =
    -2.931352998697805

nu0_parab =
    -2.566178918078498

nu0_hyp =
    -2.050947611517740
```

```
t0_elps =
    -25.575628255228040

t0_parab =
    -11.595887309505365

t0_hyp =
    -9.134385075133975
```

```
nuf_elps =
    2.900829501272578

nuf_parab =
    2.765908313733543

nuf_hyp =
    2.281185752741264
```

```
deltaNu_elps =
    5.832182499970384
```

```
deltaNu_parab =
    5.332087231812041
```

```
deltaNu_hyp =
    4.332133364259004
```

Final Position [LU] and Final Velocity [LU/TU]

```
rfMag_elps =
    7.765957289767456

rfMag_parab =
    18.180588207151615

rfMag_hyp =
    28.606931113489377
```

```
rfVec_elps = 1×3
    1.880329662676218     7.515477867864366     0.540412253555647

VfVec_elps = 1×3
    0.224072284961409     0.044678494466603     0.037807949258310

rfVec_parab = 1×3
   -1.751471165597895    16.540126925699884    -7.341003854776087

VfVec_parab = 1×3
    0.021179980079786     0.314212938110702    -0.104062864739653

rfVec_hyp = 1×3
  -21.920276273276500     7.154811025783886   -16.931233712298681

VfVec_hyp = 1×3
   -0.452494366209208     0.209326438813907    -0.372618752591200
```

F and g functions

```
f_elps =
    0.704758574576050

fdot_elps =
    0.030801441524339

g_elps =
    -16.434632102040325

gdot_elps =
    0.700650773397150
```

```
f_parab =
    -5.010767314223918

fdot_parab =
    -0.067963244487783

g_parab =
    -1.034908122774439e+02

gdot_parab =
    -1.603261710885373
```

```
f_hyp =
    -12.748840893084221

fdot_hyp =
    -0.278578792808081

g_hyp =
    -1.238327018427318e+02

gdot_hyp =
    -2.784344466073549
```

Final Energy [LU^2/TU^2]

```
energy_elps =
   -0.101950127000191

energy_parab =
       0

energy_hyp =
       0.158750158750238
```

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

```
energyF_elps =
   -0.101950127000191

energyF_parab =
     -3.469446951953614e-17

energyF_hyp =
       0.158750158750238
```

# Numerical Integration

dx [LU]


```
dx_elps = 6×1
    0.100000000000000
   -0.200000000000000
    0.010000000000000
   -0.010241945725822
   -0.012290334870986
   -0.002048389145164

dx_parab = 6×1
   -0.225163614940536
   -0.450327229881072
    0.022516361494054
   -0.010241945725822
   -0.012290334870986
   -0.002048389145164

dx_hyp = 6×1
   -0.337745422410804
   -0.675490844821608
    0.033774542241080
   -0.010241945725822
   -0.012290334870986
   -0.002048389145164
```
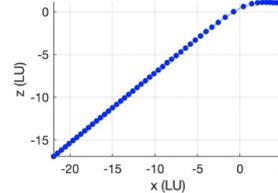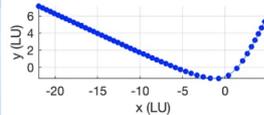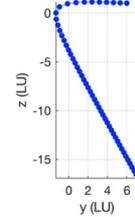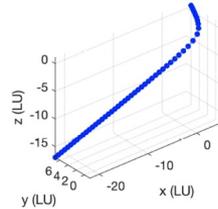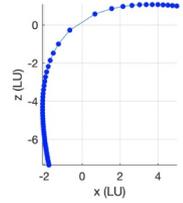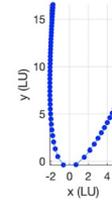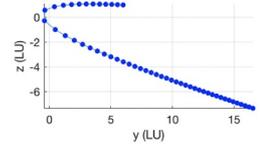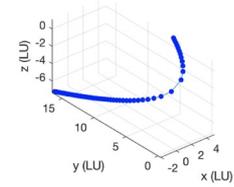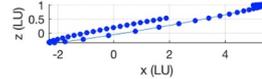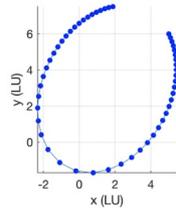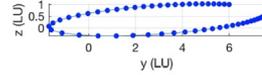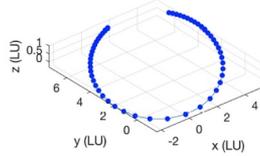
Comparing final position [LU] and final velocity [LU/TU]

rfVecs_table = 3×6 table

	Pt 1 Ellipse	Pt 2 Ellipse	Pt 1 Parabola	Pt 2 Parabola	Pt 1 Hyperbola	Pt 2 Hyperbola
1	1.880329662676218	1.880329662676146	-1.751471165597895	-1.751471165598231	-21.920276273276500	-21.920276273276745
2	7.515477867864366	7.515477867864538	16.540126925699884	16.540126925700136	7.154811025783886	7.154811025783685
3	0.540412253555647	0.540412253555640	-7.341003854776087	-7.341003854776349	-16.931233712298681	-16.931233712298770

VfVecs_table = 3×6 table

	Pt 1 Ellipse	Pt 2 Ellipse	Pt 1 Parabola	Pt 2 Parabola	Pt 1 Hyperbola	Pt 2 Hyperbola
1	0.224072284961409	0.224072284961408	0.021179980079786	0.021179980079778	-0.452494366209208	-0.452494366209213
2	0.044678494466603	0.044678494466616	0.314212938110702	0.314212938110715	0.209326438813907	0.209326438813903
3	0.037807949258310	0.037807949258311	-0.104062864739653	-0.104062864739662	-0.372618752591200	-0.372618752591203

```
diff_rfMag_elps =
      1.869308654679283e-13

diff_rfMag_parab =
      4.951178961064293e-13

diff_rfMag_hyp =
      3.290482289321648e-13
```
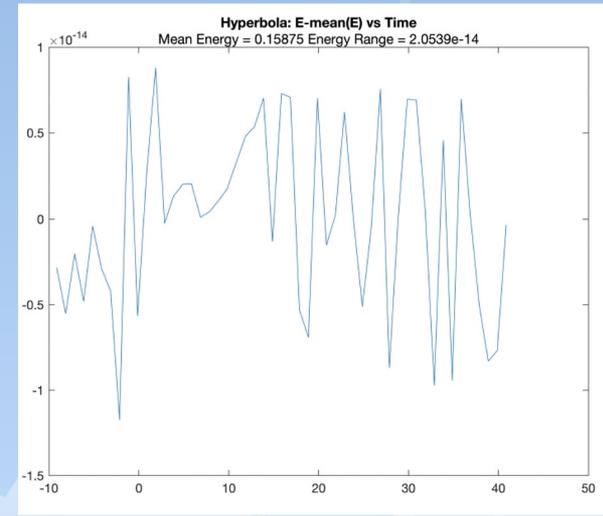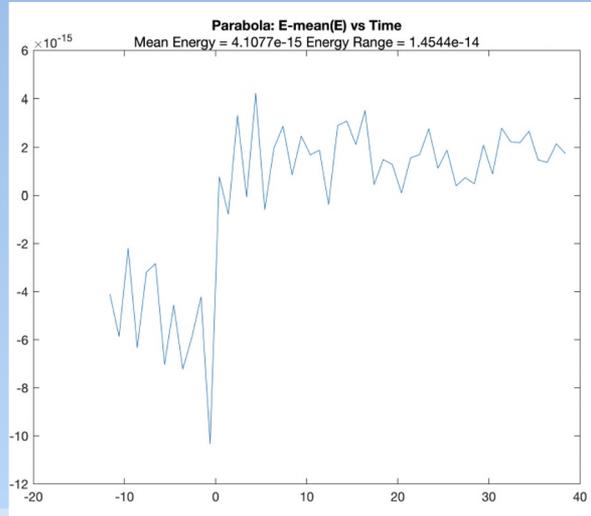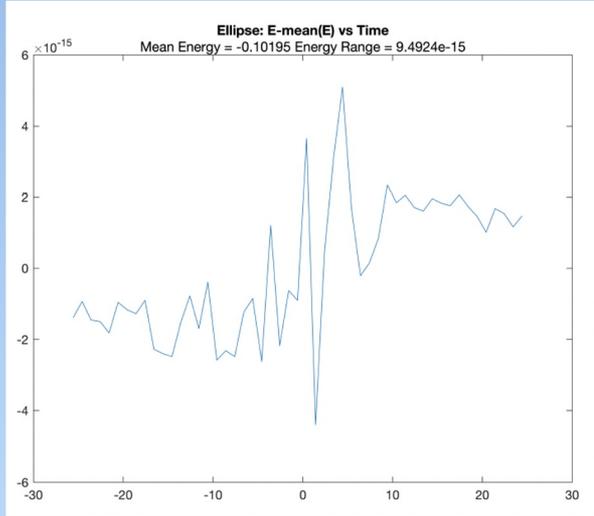
```
diff_VfMag_elps =
      1.327117835353266e-14

diff_VfMag_parab =
      1.732555555791137e-14

diff_VfMag_hyp =
      7.306414754891975e-15
```
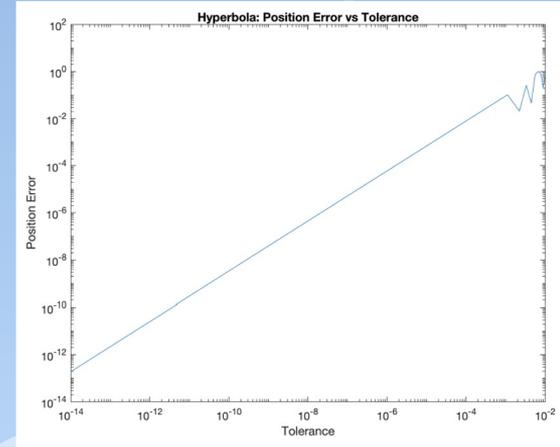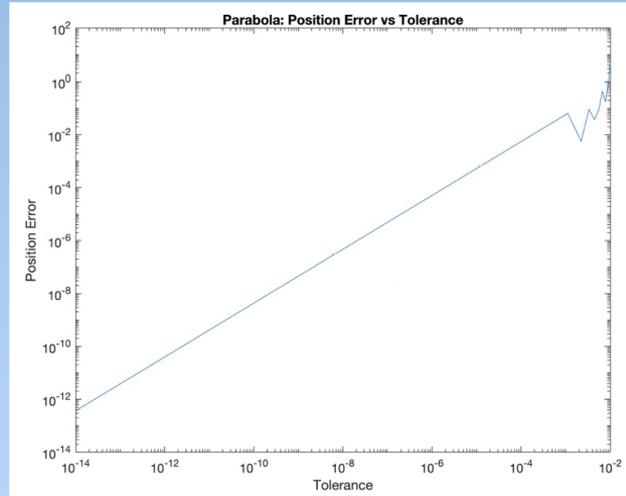
Error Comparison

Ellipse: E-mean(E) vs Time
Mean Energy = -0.10195 Energy Range = 9.4924e-15

Parabola: E-mean(E) vs Time
Mean Energy = 4.1077e-15 Energy Range = 1.4544e-14

Hyperbola: E-mean(E) vs Time
Mean Energy = 0.15875 Energy Range = 2.0539e-14

Position error vs Tolerance

Ellipse: Position Error vs Tolerance

Parabola: Position Error vs Tolerance
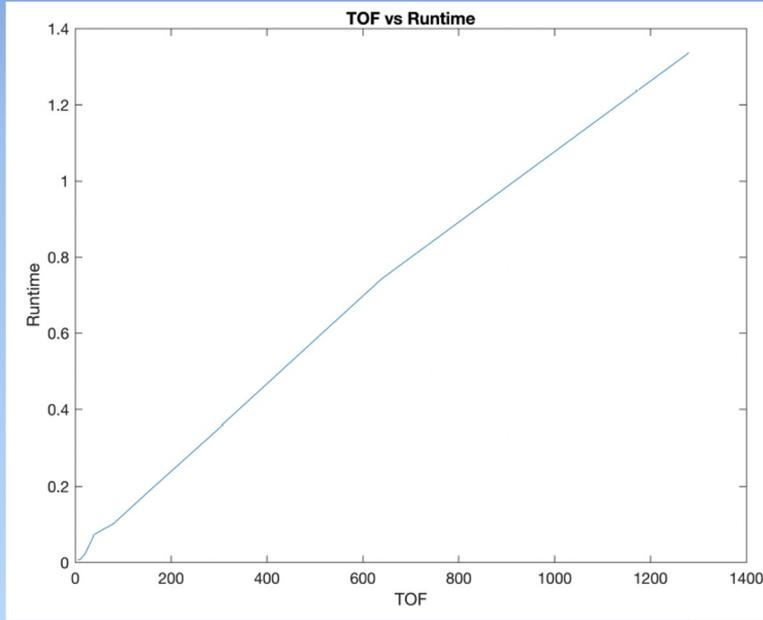

Hyperbola: Position Error vs Tolerance
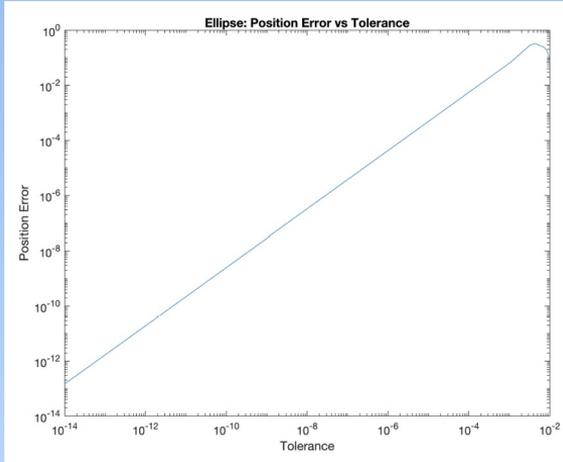
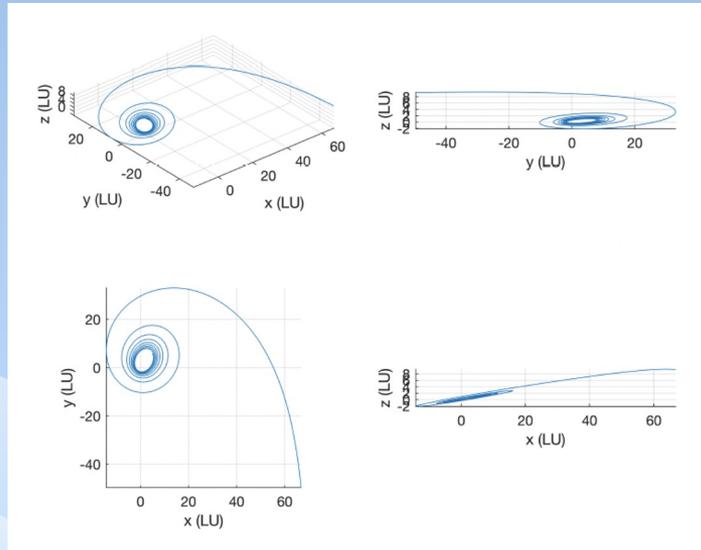# Benefits of Numerical Approach
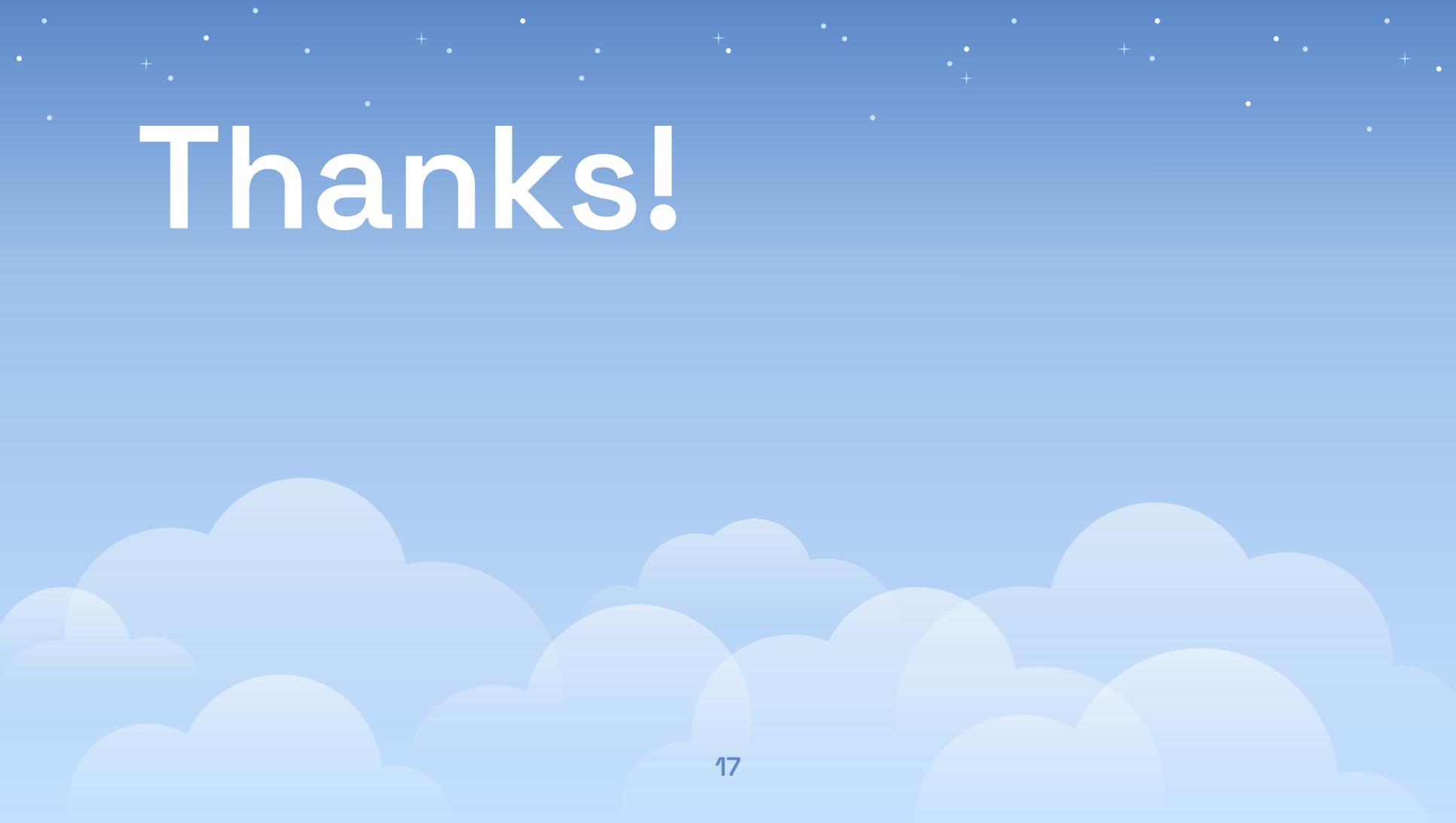

TOF vs Runtime

```
pt1_runtime =
    2.931880548000000
```

```
tEsc =
    -7.575628255228040
```

Thanks!